# Self-Adaptive Differential Evolution Based Multiple Model Variable Particle Filter for Trajectory Tracking

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## Abstract

In fixed rate state space models are the conventional models used to track the maneuvering objects. In contrast to fixed rate models, recently introduced variable rate particle filter (VRPF) is capable of tracking the target with a small number of states by imposing a Gamma distribution on the state arrival times while the object trajectory is approached by a single dynamic motion model.. It cannot estimate the position of targets in high maneuvering regions. Thus, multiple model variable rate particle filter (MM-VRPF) is utilized to overcome this shortage using various dynamic models. A weak point of particle filter is a phenomenon called degeneracy which even exists in MM-VRPF structure. In this study differential evolution method is exploited to improve the mentioned method and a novel structure called multiple model variable rate particle filter with sef-adaptive differential evolution (MM-VRPF with S-ADE) is introduced. The simulation results, particular in bearing only tracking achieved from a maneuvering target, revealed that the proposed structure has better performance while it maintains advantages of variable rate structure.

**Keywords:** target tracking; multiple model variable rate particle filter; self-adaptive differential evolution

## 1. INTRODUCTION

Generally tracking is referred to obtaining kinematic parameters of a target during a time interval based on noisy observations. During last decade tracking maneuvering targets have experienced increasing progress and has attracted great attention owing to development of numerical techniques [1]. Estimation in nonlinear systems is a prominent issue in many applications. Bayesian filter is one of the most popular estimation techniques. From its perspective the objective is to estimate a stochastic process based on noisy observations. Since this filter does not have a closed solution, different methods have been proposed for its implementation in accordance with process and measurement model. With this regard for limited linear dynamic systems grid based filters are utilized [2]. Furthermore in case of a nonlinear system and Gaussian noise Extended Kalman Filter (EKF) is exploited [3]. Increase in nonlinearity of the system, the estimation results are distorted and the posterior probability function violates Gaussian state and destroys the estimations [3],[4]. Another practical solution for implementation of Baysian filters is using nonparametric methods among which the most important one is particle filter [5]. In the above mentioned filter posterior probability density function is estimated by a set consisting of weighted particles [3],[4].

In standard methods for target tracking and particularly in particle filter, the state sampling rate is determined proportionate to measurement rate. A modern and economic approach is utilizing variable rate particle filter (VRPF) where state arrival times (new states) are modeled as pseudo Markovian random process. Although this structure would be able to track different features of motion using linear curvilinear motion dynamic model, it is not capable of providing a precise estimation in regions with high maneuver. To address this problem a structure with multiple models might be employed which models target motion dynamics using a set of models and it is able to switch between these models. The modified structure is called multiple mode variable rate particle filter (MM-VRPF). Using this method continuous certain process proposed in [6],[7] will be maintained; meanwhile, they would be adapted to variable rate structure with multiple models, so that the tracking operation is improved.

The most essential weak point which must be taken into considerations in particle filters is degeneracy phenomenon which results from increase in variance of weights [8]. In practice it has been observed that most of samples have normalized weight close to zero after a short time and only one sample has large weight. So the weights of some samples are calculated whereas they have negligible effect on final estimation which is a waste of power. To address this issue resampling is utilized. In resampling stage weighted samples at the end of a step are sampled N times. The chance of each sample for being selected depends on its weight. As a result, in this step samples with greater weights are copied several times and samples with smaller weights would be eliminated. At the end of this step a non-weighted estimation of joint posterior distribution is achieved. Numerous algorithms have been proposed for resampling; in [9] a good comparison is presented. Resampling method improves degeneracy [10]; however, it has a crucial weakness called sample impoverishment. It is due to repeat of samples with large weight. It causes all samples to have the same history after a specific time step. In this paper differential evolution optimization algorithms are utilized to mitigate degeneracy an a new set of filters are introduced; multiple model variable rate particle filter with differential evolution (MM-VRPF with S-ADE). In this algorithm particles are optimized using self-adaptive differential evolution algorithm and they are combined with the random set obtained by probability distribution in variable rate structure so that better solution is derived.

The simulation results illustrated that proposed structure increases efficiency and precision in path estimation compared to MM-VRPF.

## 2. VARIABLE RATE SAMPLING AND MODELING MOTION

### 2.1 Variable Rate Particle Filter

This paper focuses on improving tracking operation and increasing estimation precision in MM-VRPF. In order to understand other sections here a brief review on structure of variable rate particle filter is provided. More details might be found in [7],[11].

In standard constant rate stat-space models a state variable $x_t$ is defined which evolves during time with t index. The generic model is considered between time {0 and T}. variable state sequence follows a Markovian process and they are generated based on density function shown in equation (1) [6],[7].

$$x_k \sim p(x_k | x_{k-1}), \qquad \tau_k \rangle \tau_{k-1} \tag{1}$$

Where $x_k$ is State with variable rate is defined in the form of $x_k = (\tau_k, \theta_k)$ , k is a discrete index and $\tau_k$ and $\theta_k$ respectivelydenote new state arrival in i[th] state and a vector of target parameters.

In a variable rate model state assignment is not synchronous with observations. Thus, the optimum solution is when state positions (new states) are dependent on probability function. As a matter of fact, it is assumed that a observation is independent of all data points except neighboring points. Similarity probability function for consecutive values of t could be defined as equation (2) [7].

$$p(\mathrm{y}_t | x_{0:\infty}) = p(\mathrm{y}_t | x_{N_t}) \tag{2}$$

Where $x_{N_t} = \{x_l; \mathrm{k} \in N_t(x_{0:\infty})\}$ and $y_t$ is the observation.

It is noteworthy that includes all states close to observations at times t. A process in the form of $\hat{\omega}_t = f_t(\mathrm{x}_{N_t})$ is defined which might be utilized for calculating probability function. It is assumed that the largest and smallest elements of neighboring set are $N_t^+$ and $N_t^-$, respectively. Finally, common density of observations and states could be demonstrated as shown in equation (3) according to Markovian assumptions [7].

$$p(x_{0:k}, z_{0:T}) = p(x_0)\prod_{l=1}^{k} p(x_l | x_{l-1})\prod_{t=0}^{T} P(\mathrm{y}_t | x_{N_t}), \quad K \geq N^+{}_T \tag{3}$$

Where $K \geq N_T^+$ guarantees a complete neighborhood for calculating density observed at the end of T time index.

Defining $z_{0:t} = (z_0, ..., z_t)$ as observation and $x_{0:\mathrm{N}^+{}_t} = (x_0, ..., x_{N^+{}_t})$ as desired target states (which is always a random variable), it can be said that at each time step t, VRPF structure will result in an estimation of optimized filtering distribution. It is denoted (as shown by equation (2) [7]) in the form of a combination of $N_p$ multi dimensional Dirac delta each of which illustrates a particle.

$$p(X_{o:N^+_t}, N^+_t | z_{0:t}) \approx \sum_{i=1}^{N_d} \omega^i_t \sigma(x^i_{0:N^+_t} - x_{0:N^+_t}) \quad (4)$$

Where $\omega^i_t$ is the weight of i[th] particle. The above equation calculates at state arrival time by performing updating operation based on equation (5) [7] and $w^i_{t-1}$.

$$\omega^i_t \propto \omega^i_{t-1} \frac{p(y_t | x^i_{N_t}) p(x^i_{N_{t-1}+1:N^+_t} | x^i_{N^+_{t-1}})}{q(x^i_{N^+_{t-1}+1:N^+_t} | x^i_{N^+_{t-1}}, y_{0:t})} \quad (5)$$

As mentioned before in a conventional variable rate particle filter merely one model is exploited to estimate position of the target. According to [6] a CL model would be an appropriate choice in such filter for modeling target motion.

### 2.2 Multiple Model Variable Rate Particle Filter

In standard VRPF method new state arrival time and target motion are configured using a united model. However, during a maneuver motion parameters and arrival times are diverse due to the nature of targeting problem. On this basis usually state arrival times and target maneuver parameters are not estimated with a unique model. To improve the structure a multiple model variable rate structure is proposed. In this structure arrival times and maneuver parameters are modeled by a model consisting of a triplet set of parameters which improves targeting operation. In this method another state variable $(m_k)$ is added to state vector of VRPF. It shows dynamic motion mode and is denoted by equation (6) [7].

$$x_k = [\tau_k, \theta_k, m_k] \qquad m_k \in [1, ..., \mathrm{j}] \quad (6)$$

Where j means all states. Each targeting plan or program is used for demonstration of a set of dynamic states. Each state particularly demonstrates a specific feature of the target maneuver. In this paper we deal with a model consisting of 3 states so j is selected to be 3. It is worth mentioning that increase in number of states does not necessarily lead to improvement of filter performance. Thus, selecting the states in a multiple model system must be a function of desired complexity [7]. The desired structure is demonstrated in equation (7) [7].

$$\begin{aligned} p(x_k | x_{k-1}) &= p(\theta_k | \theta_{k-1}, \tau_k, \tau_{k-1}, m_k) \\ &\times p(m_k | m_{k-1}) \, \mathrm{p}(\tau_k | \tau_{k-1}, m_{k-1}) \end{aligned} \quad (7)$$

Where $p(m_k | m_{k-1})$ is the probability of state transition. It shows the probability of transition from one state to another and staying in a specific state. These probabilities are demonstrated by state transition matrix p as shown in equation (8) [7].

$$p=\begin{bmatrix} p_{11} & \cdots & p_{1r} \\ \vdots & \ddots & \vdots \\ p_{r1} & \cdots & p_{rr} \end{bmatrix} \quad (8)$$

Where $p_{hl},\{\mathrm{h,l}\}\in\{1,\ldots,\mathrm{r}\}$ demonstrates values of probability of transition from $h$ to $l$.

Practically matrix p is directly determined based on desired target maneuver. In [14] some specific methods for selecting p are mentioned. Moreover, some methods for online calculation of p matrix could be found in [13].

Consequently for target motion kinematic vector $\theta_k$ can be illustrated by $\left[L_{P,K}\quad L_{T,K}\quad V(\tau k)\quad \psi(\tau k)\quad z(\tau k)\right]$ [7] where $L_{P,K}$ and $L_{T,K}$ could be denoted in the form of Gaussian distribution [7].

Furthermore, $p(\tau_k|\tau_{k-1},m_{k-1})$ in equation (7), is conditional to discrete variable mode. The previous arrival time is shown by a shifted Gamma distribution [7].

Now a combination of multiple model structure with variable rate models is presented. Similar to standard structure in the multiple model variable rate structure mentioned steps are taken to estimate the state.

- Initial setting

In this stage values are assigned to all particles according to a determined distribution. At time t=0, $N_p$ samples are selected; then, selected samples are weighted based on their similarity to actual value in the form of equation (9).

$$w_{t=0}^{i}=\frac{1}{N_p},\quad i=1,\ldots,N_p \quad (9)$$

Where $w_{t=0}^{i}$ is particle weight at time t=0.

- Propagation step

In this step as soon as new state arrives, $N_p$ samples are selected based on q(.) distribution which plays the role of previous state distribution. q(.) distribution is stated as shown by equation (10) [7].

$$q(x_{N_{t-1}^{+}+1:\mathrm{N}_t^{+}}\left|x_{N_{t-1}^{+}}^{i},y_{o:t}\right.)=p(x_{N_{t-1}^{+}+1:N_t^{+}}\left|x_{N_{t-1}^{+}}^{i}\right.) \quad (10)$$

- Updating the particle weights

Updating is performed based on simplified form of equation (5). In this equation if previous distribution q(.) is utilized, a simpler equation in the form of equation (11) is derived [7] which is exploited for calculating particle weights $w_t^i, i=1,\ldots,\mathrm{N}_P$.

$$w_t^i\propto w_{t-1}^i p(y_t\left|x_{N_t}^i\right.) \quad (11)$$

Thus, $p(y_t|x^i_{N_t})$ probability in updating operation of $\omega^i_t$ could be defined in the form of equation (12).

$$p(y_t|x_{N_t}) = p(y_t|\widehat{\omega}_t) \quad (12)$$

Finally, it could be concluded that posterior probability function $p(x_{N_t}, N^+_t | y_{0:t})$ is estimated by associated weight vectors $\{y^i_t\}^{N_p}_{i=1}$ and particles $\{x^i_{N_t}\}^{N_p}_{i=1}$. Afterwards, if $\hat{N}_{eff} = 1/\sum_{i=1}^{N_p}(w^i_t)^2$ is less than default threshold value, resampling operation will be done.

MM-VRPF structure does not need regeneration stage whereas it is necessary in VRPF framework; thus, it considerably reduces computational load [7].

### 3.3 Multiple Model Variable Rate Particle Filter With Self-Adaptive Differential Evolution

Degeneracy phenomenon is a weakness of particle filter. This phenomenon is resulted from variance of sample weights and it is still a problem in MM-VRPF structure.

Many efforts have been made to generate a new group of particles which are able to generate higher weights such that these particles are substituted for particles with much smaller weights. In this paper, self- adaptive differential evolution algorithm [14] is exploited to obtain such particles. Differential Evolution (DE) has been shown to be a powerful evolutionary algorithm for global optimization in many problems. Self-adaptation has been found to be high beneficial for adjusting control parameters during evolutionary process, especially when done without any user interaction [16]. These particles have the most proper unique values. With this regard, fitness function in S-ADE is a function for calculating the weight of a particle.

## 4. ALGORITHM

In Fig. 1 the result of merging DE and MM-VRPF algorithm is presented.

Input: Initialization

1: Set t = 0

2: For $i = 1 \leq N_p$, $x^i_0 \sim p(x_0)$ draw equally weighted samples

from the predefined.

**initial state**

4: distribution and set t = 1 with optimal states which is calculated

by Self adaptive differentia Evolution.

**Propagation step**

5: Set $k = N + t$

6: for $i = 1 \leq N_p$

7: - While the neighborhood $N^i_t$ is incomplete

8: * Set k = k + 1 and draw samples form the proposal distribution

9: $x_k^i \sim q(x_k | x_{k-1})$ until $\tau_k \geq t$.

10: /// The particles are optimized by Differential evolution.

11: /// Mixing particles which is achieved from two pervious steps with the parameter $N_d$.

**Weight update step**

12: Calculate the particle weights

$$w_t^i \propto w_{t-1}^i \frac{p(y_t | x^i{}_{N_t}) p(x^i{}_{N_{t-1}+1:N^+{}_t} | x^i{}_{N^+{}_{t-1}})}{q(x^i{}_{N^+{}_{t-1}+1:N^+{}_t} | x^i{}_{N^+{}_{t-1}}, y_{0:t})}$$

13: Normalize the particle weights.

**Resampling step**

14: Resample $\{x_{N_t}^i, w_t^i\}_{i=1}^{N_p}$ if effective sample size

$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N_p} (w_t^i)^2}$, is Below a preset threshold.

15: Set $t = t + 1$

16: **Iterate through Propagation step**

Fig. 1. MM-VRPF with S- ADE

The difference between proposed mechanism and MM-VRPF algorithm is that it selects a number of particles based on differential evolution algorithm instead of applying a probability distribution. Subsequently, $N_d$ optimized samples are combined with other remained samples derived from probability distribution and constitute a set of optimized samples which can be utilized in next steps of algorithm structure. In other words, samples are optimized by differential evolution algorithm. Then, they are combined with random set obtained by probability distribution so that samples result in better response.

## 5. SIMULATION

In this section a comparison between proposed method and VRPF and MM-VRPF methods is performed. The practical application of our method in trajectory tracking of maneuvering target specially its bearing-only will be investigated. On this basis, for observation y at time t equation (13) can be written [15].

$$y_t = \arctan(\frac{l_1 - l_{10}}{l_2 - l_{20}}) + v_t \quad (13)$$

Where arctan(.) demonstrates nonlinear relationship, is sensor noise and $[l_{10} - l_{20}]^T$ denotes the position of sensor [15].

A. *scenario*

In Fig. 2 desired trajectory is depicted. According to mentioned scenario in [15] target and observer start their motion from origin with constant velocity of 4 and 5 knotes, respectively and courses of -150 and 140, respectively. After that, the target executes a maneuver with constant turn rate 24$^o$/min between 20 and 25 minutes. Finally, the same course will be maintained till the end. After movement, observer experiences a maneuver in (12-16) time interval with constant turn rate of $30^\circ / \min$ to reach 20 course. For this scenario it is assumed that the total number of observations is 40 and the period of observation is 1 minute.

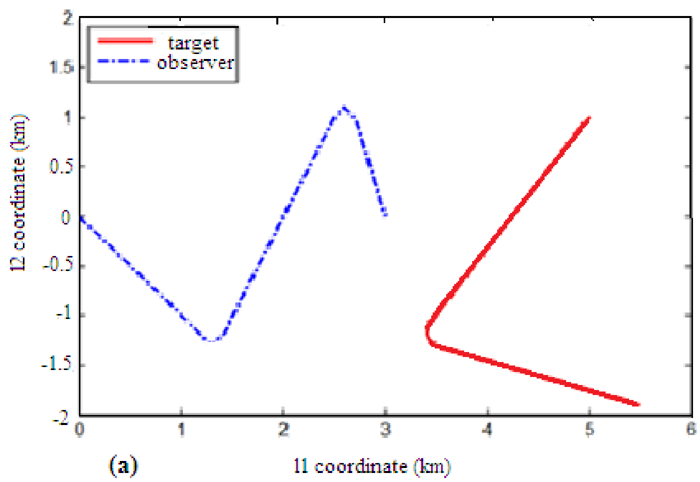


Fig 2: A true trajectory for a maneuvering target

### 4.1 *Simulation results and analysis*

There are two efficiency measures for evaluation of tracking filters; time averaged root mean square position error $(RMSE)$ and instant root mean square position error $(\mathrm{RMSE})_t$ are mentioned in equation (14). The achieved values for these two measures are obtained by Monte Carlo method with L=100 runs [7].

$$RMSE_t = \sqrt{\frac{1}{T}\sum_{i=1}^{L}(\widehat{l}_{1t}^{\,i} - l_{1t}^{\,i})^2 + (\widehat{l}_{2t}^{\,i} - l_{2t}^{\,i})^2}$$
$$RMSE = \sqrt{\frac{1}{LT}\sum_{t=1}^{T}\sum_{i=1}^{L}(\widehat{l}_{1t}^{\,i} - l_{1t}^{\,i})^2 + (\widehat{l}_{2t}^{\,i} - l_{2t}^{\,i})^2} \quad (14)$$

Where T is the index to the last observation. Moreover, for executing i$^{th}$ run $\widehat{l}_t^{\,i}$ and $l_t^{\,i}$ values respectively state estimated and actual positions at time t.

To observe the behavior of mentioned methods in description of target motion, initial conditions are set using actual values. For instance, Gaussian value with $(\sigma_\theta = 1.5)$ for bearing and $(\sigma_r = 100m)$ for range are chosen. Additionally, considering unique features such as displacement parameters and velocity of the target, the values of P transition matrix are selected as shown in equation (15) [7],[15].

$$P = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.35 & 0.45 & 0.2 \\ 0.35 & 0.2 & 0.45 \end{bmatrix} \qquad (15)$$

Table 1 represents sojoum time distribution parameters for MM-VRPF and VRPF [9]. This table considers 3 states; (n=1) for modeling direct form of motion and (n=2,3) to model motion maneuvers of target. The multiple model structure is able to switch between these states.

TABLE 1: MM-VRPF and VRPF parameters for the desird scenario.

| | **MM-VRPF** | | **VRPF** |
|---|---|---|---|
| | ***MOD.1*** | ***MOD.2,3*** | |
| $\mu_{T,n}, \sigma_{T,n}$ | (0,100) | (0,100) | (0,100) |
| $\mu_{p,n}, \sigma_{p,n}$ | (0,500) | (±1100,3000) | (0,5000) |
| $\alpha_n, \beta_n$ | (1.5,4) | (0.5,0.35) | (0.5,6.5) |
| $\tau_n$ | (0) | (0.5) | (0) |

In Fig. 3 sections (a), (b) and (c) explain state arrival time and trajectory points generated by the VRPF, MM-VRPF and MM-VRPF with S-ADE respectively. Fig. 3(b) and 3(c) show the MM-VRPF and MM-VRPF with S-ADE structre are capable of locating frequent states at bearing region while using a parsimonious state representation for the smooth regions of the trajectory.

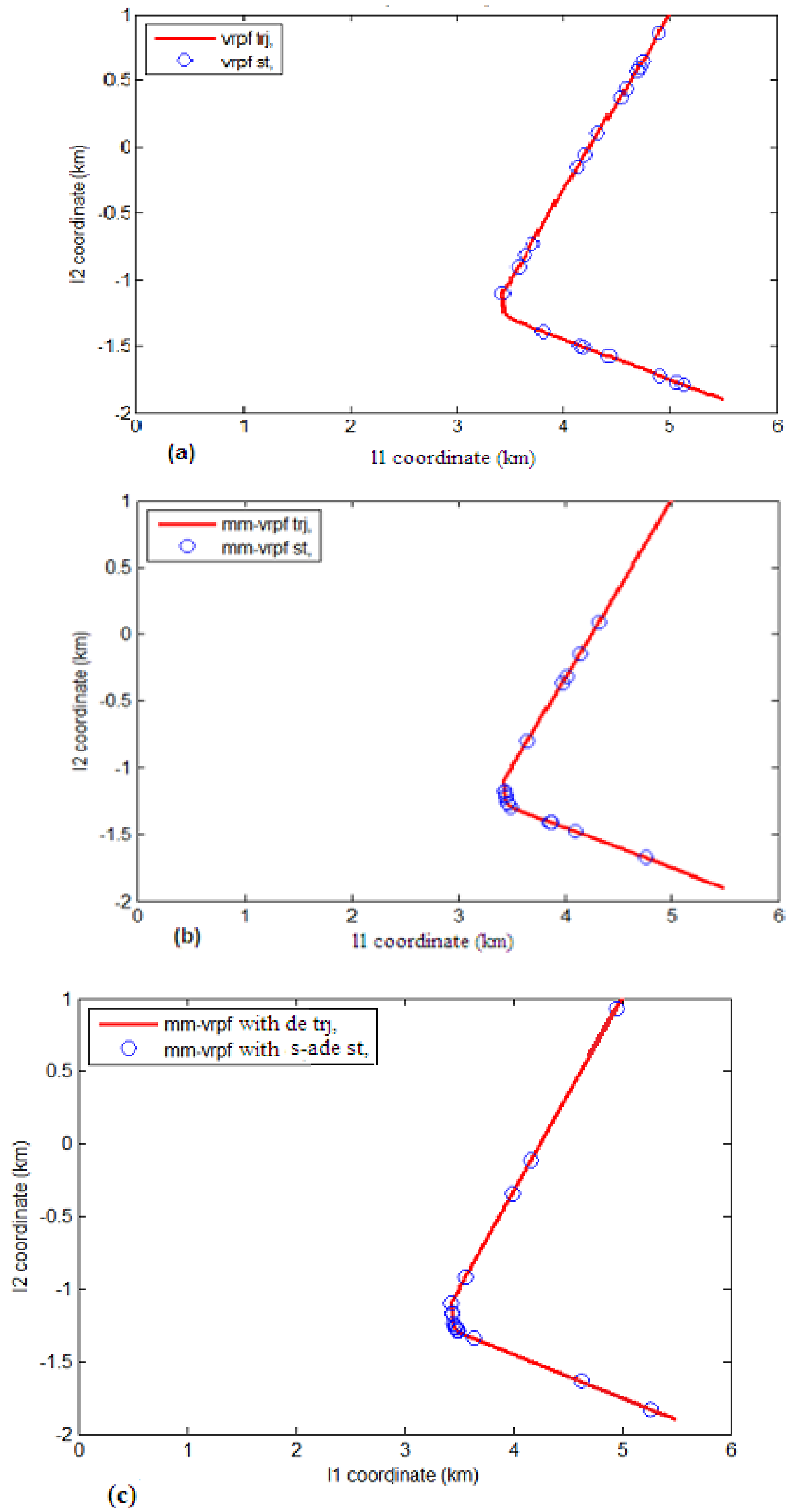


Fig 3: Trajectories and states of a particle generated by (a) the VRPF, (b) the MM-VRPF and (c) the MM-VRPF with S-ADE.

Fig. 4 shows $RMSE_t$ values for 3 mentioned structure. In Fig. 4(a) the diagram of $RMSE_t$ versus $N_p = 2000$ and in Fig. 4(b) versus $N_p = 8000$ are shown using 40 observations for mentioned scenarios. In addition, numerical values of $RMSE$ for $N_p = 2000$ and $N_p = 8000$ could be seen in table 2. Investigating values presented in Table 2 together with $RMSE_t$ diagram in Fig. 4, the superiority of proposed structure could be concluded.

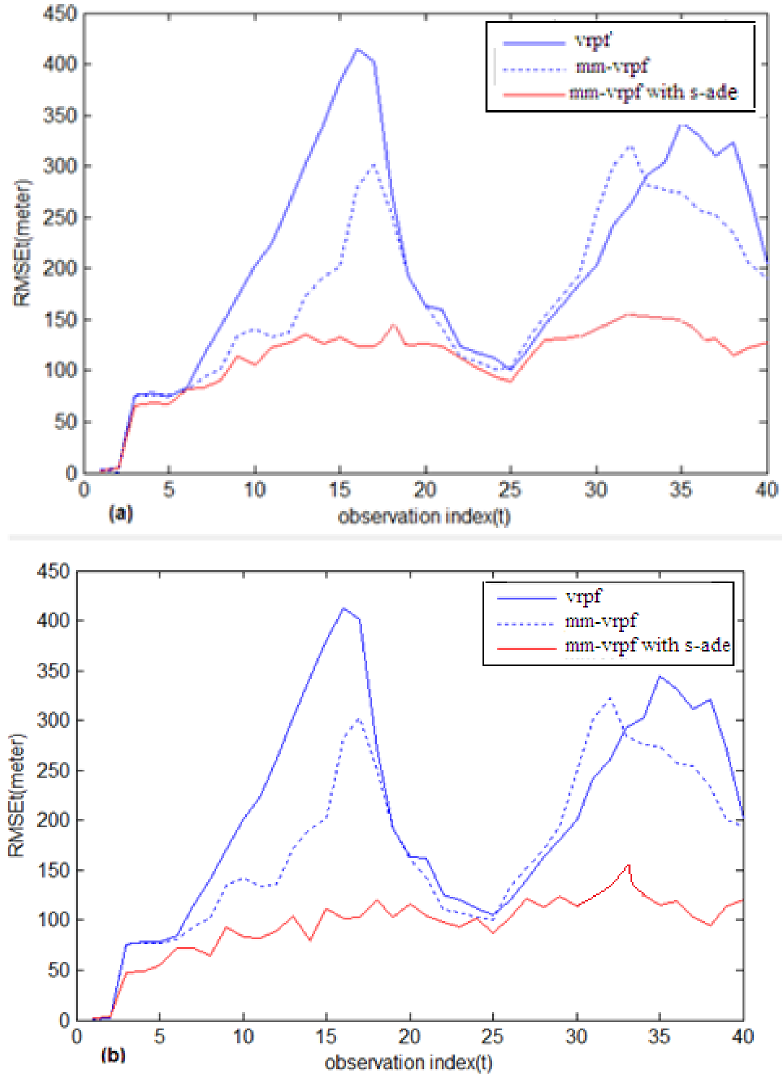


Fig 4: $RMSE_t$ versus time t for true initials by (a) $N_p = 2000$, (b) $N_p = 8000$.

TABLE 2. RMSE for varying particle size obtained by using true initials for the desird Scenario.

| | ***VRPF*** | ***MM-VRPF*** | ***MM-VRPF with S-ADE*** |
|---|---|---|---|
| $N_p = 2000$ | 204.12 | 190.26 | 126.86 |
| $N_p = 8000$ | 201.17 | 184.67 | 119.56 |

## 6. CONCLUSION

In this paper a new approach to merging S-ADE structure with MM-VRPF, is proposed. In this approach MM-VRPF with S-ADE structure is suggested to improve degeneracy phenomenon which is a consequence of increase in sample weights. This method combines optimized samples generated by S-ADE algorithm with other samples obtained from probability distribution applied to variable rate structure. As a result an optimized set of samples is achieved which will be used for estimation. The simulation results revealed relative superiority of this method in tracking high maneuver points of targets.

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